PHYS 100C, LECTURE #9:

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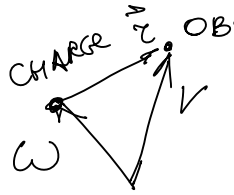
$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qc}{rc-\vec{r}\cdot\vec{v}} \qquad A(r,t) = \frac{v}{c^2}V(r,t)$$

$$E = -\nabla V - \frac{\partial A}{\partial t} \qquad B = \nabla\times A$$



$$\vec{r} = \vec{r} - \vec{\omega}$$

$$t - t_R = \frac{|\vec{r}-\vec{\omega}|}{c}\left(=\frac{|r|}{c}\right)$$

$$\nabla V = \frac{qc}{4\pi\varepsilon_0}\frac{(-1)}{(rc-\vec{r}\cdot\vec{v})^2}\nabla(rc-\vec{r}\cdot\vec{v})$$

$$\nabla(rc) = c\cdot\nabla r = -c\nabla t_R$$

$$\nabla(\vec{r}\cdot\vec{v}) = \underbrace{(r\cdot\nabla)v}_{\#1} + \underbrace{(v\cdot\nabla)r}_{\#2} + \underbrace{r\times(\nabla\times v)}_{\#3} + \underbrace{v\times(\nabla\times r)}_{\#4}$$

#1: $(r\cdot\nabla)v = r_x\cdot\frac{\partial v}{\partial x} + \ldots$

$$r_x\cdot\frac{\partial v}{\partial x} = r_x\cdot\underbrace{\frac{\partial v}{\partial t_R}}_{a}\cdot\frac{\partial t_R}{\partial x}$$

$$(r\cdot\nabla)v = a(\vec{r}\cdot\nabla t_R)$$

#2 $(v\cdot\nabla)r = (v\cdot\nabla)(r-\omega) = (v\cdot\nabla)r - (v\cdot\nabla)\omega$

$$(v\cdot\nabla)r = v_x\cdot\frac{\partial\vec{r}}{\partial x} + \ldots = v_x\cdot\hat{x} + v_y\hat{y} + v_z\hat{z} = \vec{v}$$

$(v\cdot\nabla)\omega = v(v\cdot\nabla t_R)$ (see #1)

#3 $\nabla\times v = \frac{\partial v_z}{\partial y}\cdot\hat{x} + \ldots$

$$\frac{\partial v_z}{\partial y}\cdot\hat{x} = \underbrace{\frac{\partial v_z}{\partial t_R}}_{a}\cdot\frac{\partial t_R}{\partial y}\cdot\hat{x}$$

$$(\nabla\times v) = -a\times(\nabla t_R)$$

#4 $\nabla\times r = \nabla\times(r-\omega) = \nabla\times r - \nabla\times\omega$

$\nabla \times V = 0$

$\nabla \times \omega = -V \times (\nabla t_R)$ (see #3)

PLUG #1 - #4 IN:

$\nabla (r \cdot v) = a(\vec{r} \cdot \nabla t_R) + V - V(v \cdot \nabla t_R) -$
$- r \times (a \times \nabla t_R) + V \times (V \times \nabla t_R)$

$r \times (a \times \nabla t_R) = a(\vec{r} \cdot \nabla t_R) - \nabla t_R (r \cdot a)$

$V \times (V \times \nabla t_R) = V(v \cdot \nabla t_R) - \nabla t_R \cdot v^2$

$\nabla (r \cdot v) = V - \nabla t_R (r \cdot a) - \nabla t_R \cdot v^2$

$\nabla V = \frac{qc}{4\pi\varepsilon_0} \frac{1}{(rc - \vec{r} \cdot \vec{v})^2} \cdot \left(V - (c^2 - v^2 + r \cdot a) \nabla t_R \right)$

$\nabla t_R = ?$

$-c \nabla t_R = \nabla r = \nabla \left(\sqrt{\vec{r}^2} \right) = \frac{1}{2} \frac{1}{\sqrt{r^2}} \cdot \nabla (\vec{r} \cdot \vec{r}) =$

$= \frac{1}{|r|} \cdot \left[(r \cdot \nabla) r + r \times (\nabla \times r) \right]$

$(r \cdot \nabla) r = (\vec{r} \cdot \nabla) r - (r \cdot \nabla) \omega = r - V (r \cdot \nabla t_R)$

$\nabla \times r = V \times \nabla t_R$

$-c \nabla t_R = \frac{1}{|r|} \left[\vec{r} - (r \cdot \nabla) \nabla t_R \right]$

$\nabla t_R = - \frac{\vec{r}}{rc - \vec{r} \cdot \vec{v}}$

$\nabla V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qc}{(rc - \vec{r} \cdot \vec{v})^3} \cdot \left((rc - \vec{r} \cdot \vec{v}) \cdot \vec{v} - (c^2 - v^2 + \vec{r} \cdot \vec{a}) r \right)$

$\frac{\partial A}{\partial t} = \frac{1}{4\pi\varepsilon_0} \frac{qc}{(rc - \vec{r} \cdot \vec{v})^3} \left[(rc - \vec{r} \cdot \vec{v}) \left(-v + \frac{ra}{c} \right) + \frac{r}{c} (c^2 - v^2 + ra) v \right]$

$u = c\hat{r} - \vec{v}$

$$E(r,t) = \frac{q}{4\pi\epsilon_0} \cdot \frac{\hat{r}}{(\hat{r}\cdot u)^3}\left[(c^2-v^2)u + \bar{r}\times(\bar{u}\times\bar{a})\right]$$

$$\nabla\times A = \frac{1}{c^2}\nabla\times(\bar{V}\cdot v) = \frac{1}{c^2}\left[\bar{V}(\nabla\times v) - v\times(\nabla\times\bar{V})\right]$$

$$\nabla\times A = -\frac{1}{c}\cdot\frac{q}{4\pi\epsilon_0}\cdot\frac{1}{(u\cdot r)^3}\cdot r\times\left[(c^2-v^2)\bar{v} + (r\cdot a)\bar{v} + (r\cdot u)a\right]$$

$$B(r,t) = \frac{1}{c}\,\hat{r}\times\bar{E}(\bar{r},t)$$

If $v=0$ $\quad a=0$ $\quad E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}\cdot\hat{r}$

Total force on charge:

$$F = \frac{qQ}{4\pi\epsilon_0}\cdot\frac{r}{(r\cdot u)^3}\left[(c^2-v^2)u + r\times(u\times a) + \right.$$

$$\left. + \frac{v_Q}{c}\times\left[\hat{r}\times\left[(c^2-v^2)u + \vec{r}\times(\vec{u}\times\vec{a})\right]\right]\right]$$